Mathematical challenges of sustainability

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October 2010

I will discuss here models of the interaction between human populations and their environment. Most models of this interaction have two components, one for the human population and another for the environment. The level of detail in a model and the way in which it is constructed tends to reflect the background of the person making the model. Demographers, economists, and others who come at the question from the perspective of the human actors build models that are detailed in terms of one or more aspects of the human component but relatively poor in describing the environment and non-human species. The converse is often true of models constructed by ecologists and environmental scientists. I will focus here on dynamic models that relate human population change to resource dynamics.

Most human population models that are used in discussions of sustainability have their roots in some form of Malthusian regulation. Perhaps the simplest such model is the logistic. However the logistic and other simple models have little to say about the nature of population regulation, the facts of human demography, or the dynamics of resource use and renewal. However to understand questions about adaptation, control, and management we must unpack these details.

A good starting point is to think about the notion of carrying capacity, which is strikingly useless in discussions of human population. A useful discussion of the reasons why this is so, and of the actual factors that determine human population size, is given by Joel Cohen (1995). As he points out, "carrying capacity" is a shorthand for the result of interactions between demography, culture, technology, and resources. These interactions are at the core of models that address aspects of sustainability.

The simplest models that attempt to capture aspects of these interactions are generalizations of the logistic, and even these simple models pose interesting mathematical questions that are relevant to the study of sustainability. Cohen (1995) describes a toy model in which there is a feedback between human population and human carrying capacity. Over long time scales this feedback can describe innovation and technological change along the lines proposed by Ester Boserup (1965). Ronald Lee (1986,1987) has shown that such coupled models generate alternative equilibria between populations and their resources, and that trajectories of innovation move populations between equilibria. An important set of mathematical questions concerns the elaboration of the dynamic behavior of such coupled systems. In particular it would be interesting to do extend simple coupled models by considering the effects of different resource dynamics, different processes and rates of innovation, the effects of stochastic perturbations, and the possibility of multiple equilibrium states.

The equilibrium and dynamics of such relatively simple models depend on the assumption that the parameters and functional relationships in the models do not change. In this equilibria of such models can be thought of as stationary distributions in a suitable state space. However many of the questions that interest us involve long term change of two kinds. First the parameters of the models may change with time because they are linked to some larger scale environmental driver, for example climate change. Second the parameters, and possibly even the functional relationships, may evolve. On the human side this might be due to cultural evolution, including changes in behavior, innovation leading to new technology, and culture. On the resource side there may be biological evolution produced both within the natural system or as a result of selection practiced by humans. In the presence of such change the nature of equilibria also changes. Assuming that evolutionary changes of parameters or interactions take place slowly, an equilibrium is a kind of quasi-stationary distribution and we need to follow changes in the distribution over time. It is also possible that evolutionary change may occur rapidly in which case the dynamics of the system can become quite complex. There is considerable potential in extending the models developed by Cohen and Lee by incorporating cultural evolution (Cavalli Sforza and Feldman 1981), niche construction (Odling Smee et al. 2003), and biological evolution.

A different but also valuable extension of simple models makes a start on the problem of understanding sustainability from the point of view of the different actors in a system. Thus models which provide an explicit description of human demography, in particular mortality and fertility, allow us to measure aspects of human welfare which are simply not accessible in models like the logistic. Similarly, models that incorporate structural aspects of the resources, for example demography, interactions within and between species, can be used to describe aspects of the "welfare" of resources. Some work along these lines has been done by scientists studying prehistoric, or at least early, human populations (Wood 1998). In a series of models that we have developed (Lee et al. 2008,2009, Puleston and Tuljapurkar 2009) the demographic features of the human complement are described in detail, and the linkage between human demography and resources comes via the effect of consumption on birth and death rates (Bengtsson et al. 2004). The resources here are agricultural and resource model describes soil organic matter and nutrient dynamics. We include culture in the form of the choice of plants to be grown, the amount of labor allocated to agriculture, and aspects of technology such as irrigation and mulching, we can provide a fairly detailed analysis of the interaction between humans and environment. Many questions about sustainability can be studied in this context for example, the vulnerability of such a system to varying climates, and the dependence of equilibrium human population number and density on human reproductive choices.

It is also possible to study important aspects of adaptive change, by studying the consequences of different behaviors. For example, given spatially heterogeneous resources, how does human behavior in terms of resource exploitation and resource management affect human welfare and the dynamics of resources? Given temporally varying resources, how does human behavior in terms of sharing and crowding affect the average and the variance of human welfare, as measured by say life expectancy or mortality. What are the dynamic consequences of innovation, for example in the management or choice of resources or in the manipulation of resources as in the natural selection of crop varieties? In these more fully developed models we may also examine adaptive change is in terms of cultural evolution. The objective is to ask whether certain kinds of behavior, for example the control of fertility by changing inter-birth intervals or by infanticide, will increase in frequency in the population and under what conditions. A different way of looking at these questions is in terms of niche construction.

Mathematical work along the lines discussed above has considerable promise when applied to early human populations. A big strength of this work is that we can actually test the models. But of course, a big challenge is to develop such models for prehistoric populations but then extend them in ways that make sense to contemporary populations. The human component becomes substantially more complicated when we incorporate economics, especially as it affects choices, values, norms and behavior in modern societies. In principle, economic theory provides ways of incorporating some of the diversity and complexity of modern economic systems, in particular by tracking production and consumption of various kinds. But there are many challenges involved in coupling economic dynamics with human demography. Some of these challenges have to do with interactions that we know have occurred and are likely to influence the future. For example there are clear relationships between human demographic change, such as the fertility transition, and trajectories of economic growth. In in the past century and perhaps even more so in future decades, demographic change in mortality and fertility will lead to substantial effects on economic performance. Demographic changes in the human population are also likely to affect our environment and our resource use (R). Other challenges involve the different ways in which economic models and ecological models capture the behavior of their respective components. Ecological models use a dynamic approach whereas economic models tend to use a utility maximization paradigm. The coupled models I have discussed above use a dynamic approach but it would be interesting and challenging to formulate utility-based models in which we capture utility for the natural world along with utility for humans.

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